# MathsStart

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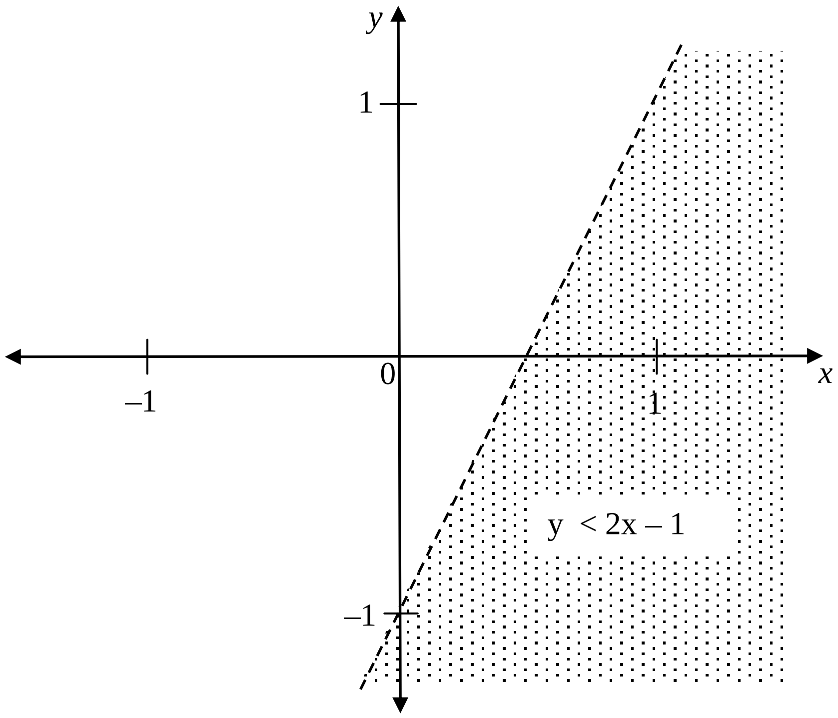
New books will be created during 2013 and 2014)

THE UNIVERSITY of ADELAIDE

## Topic 2

# Linear Functions





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# This Topic...

This module introduces linear functions, their graphs and characteristics. Linear functions are widely used in mathematics and statistics. They arise naturally in many practical and theoretical situations, and they are also used to simplify and interpret relationships between variables in the real world. Other topics include simultaneous linear equations and linear inequalities.

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## Prerequisites

You will need to solve linear equations in this module. This is covered in appendix 1.

## Contents



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# 1

# Linear Functions and Straight Lines

Linear functions are functions whose graphs are straight lines. The important characteristics of linear functions are found from their graphs.

## 1.1 Examples of linear functions and their graphs

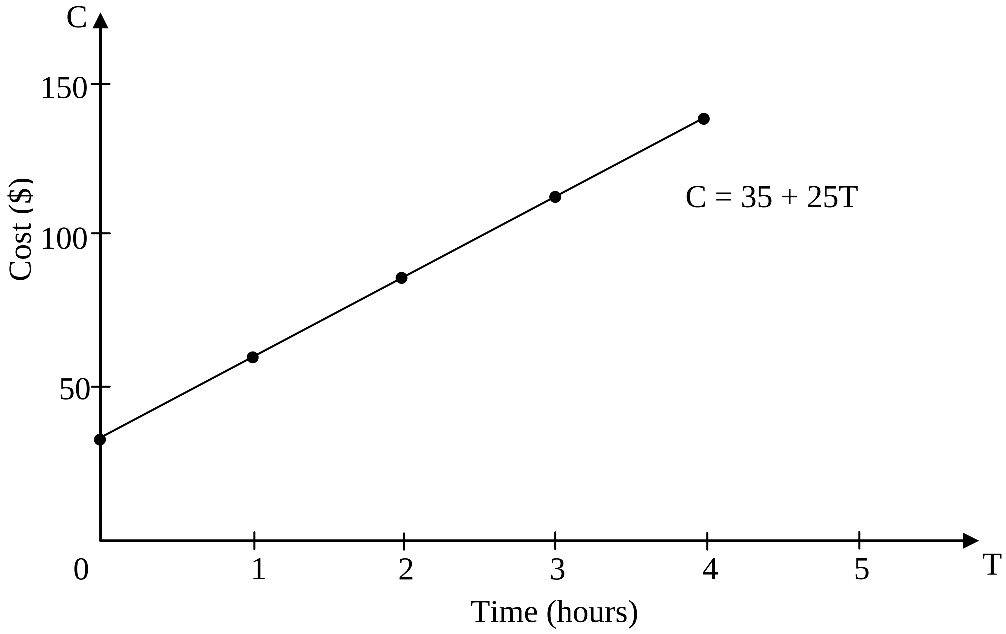
Cost functions are commonly used in financial analysis and are examples of linear functions.

*Example*

The table below shows the cost of using a plumber for between 0 and 4 hours. This plumber has a $35 callout fee and charges $25 per hour.

| **Hours** | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| **Cost** | 35 | $35+25\times 1$ | $35+25\times 2$ | $35+25\times 3$ | $35+25\times 4$ |

It can be seen from this table that the cost function for the plumber is C = 35 + 25T, where T is the number of hours worked.  This is a linear function and it has a straight line graph.

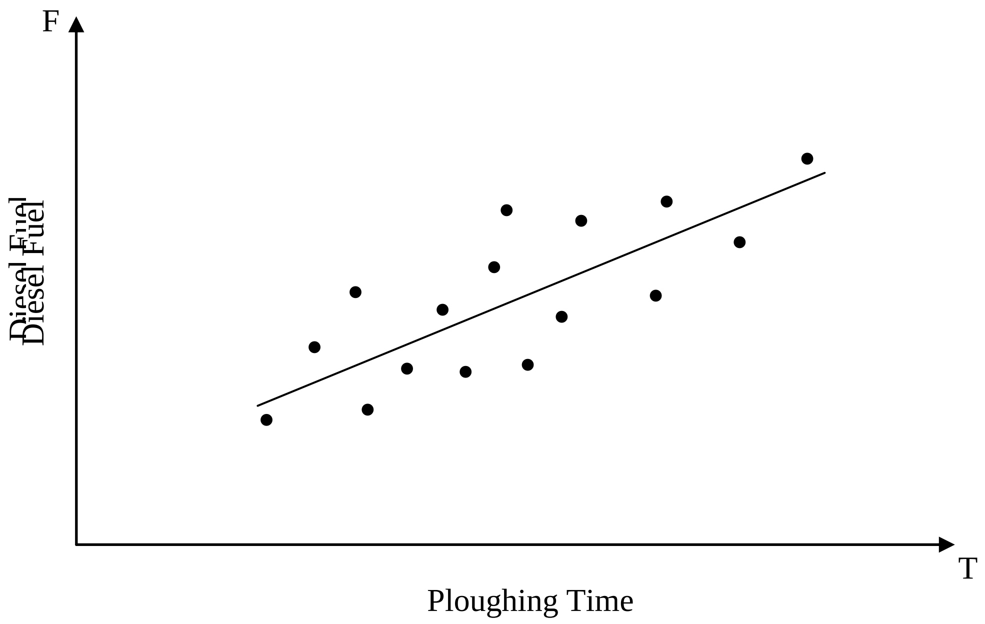



The graph cuts the C-axis at the point (0, 35), corresponding to the cost of the call out fee when T = 0. The cost then increases steadily at a rate of $25 per hour.

Lines of best fit are often used to *model* the relationship between two variables. A line of best fit is a straight line that best fits a set of data, and is usually drawn with the aid of a software package like Excel.

*Example*

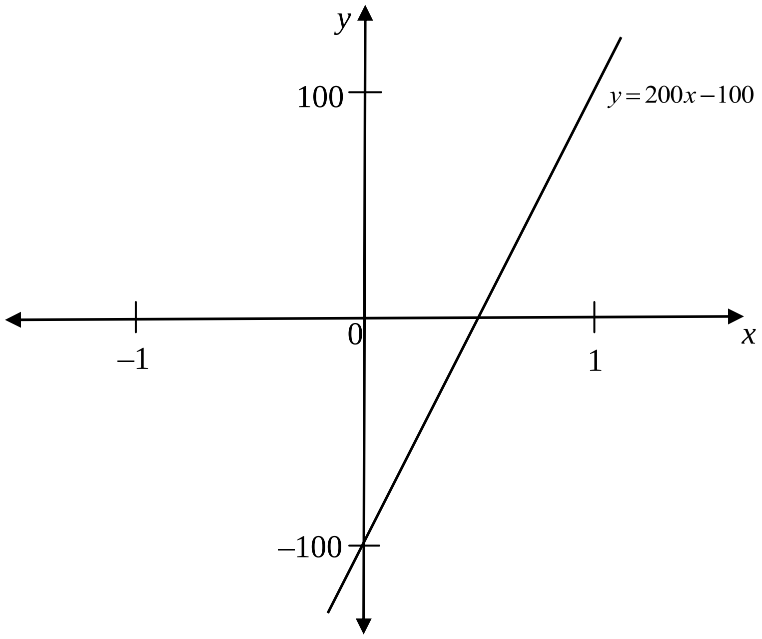
The graph below shows the amount of diesel fuel used by a farmer when ploughing a variety of paddocks at a constant speed (the data points are indicated by dots). The relationship between fuel used and time taken is roughly linear (i.e. given by a straight line). The line can be used to estimate future fuel needs and costs.




When studying the mathematical properties of general linear functions we use x to represent the independent variable and y to represent the dependent variable. We also place no restrictions on the domain of the independent variable x. The x- and y-intercepts of linear functions (ie. where the straight lines cut the x- and y-axes) are important features of linear functions, and are usually shown on their graphs unless there is a good reason not to.

*Example*

The graph of the linear function $y = 200x - 100$ is shown below. The *x*-intercept is (0.5, 0) and the *y*-intercept is (0, –100). When the value of *x* increases by 1 unit, the value of *y* increases by 200 units.




*Example* *(See Appendix: Linear Equations)*

The $x$- and $y$- intercepts of the linear function $y = 200x - 100$ can be found algebraically as follows:

(a) To find $x$-intercept:

Put $y = 0$

$200x - 100 = 0$

$200x = 100$

$x = 100/200 = 0.5$

The $x$-intercept is (0.5, 0).

(b) To find $y$-intercept:

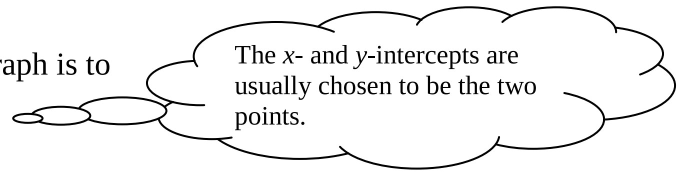
Put $x = 0$

$y = 0 - 100 = -100$

The $y$-intercept is (0, –100).

The quickest way of drawing a straight line graph is to

- find two points on the line, then
- draw a straight line through these two points.




## Problems 1.1

Each function below is a linear function, with a straight line graph. Draw each graph, showing the $x$- and $y$-intercepts.

(a) $y = 2x - 1$ (b) $y = 20x - 10$ (c) $y = 0.1(2x - 1)$

(d) $y = x$ (e) $y = 2x$ (f) $y = 3x$

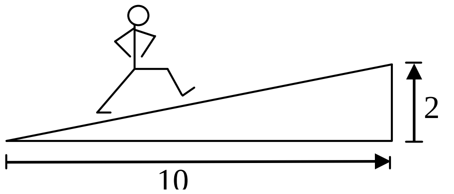
(g) $f(x) = -x + 1$ (h) $f(x) = -2x - 1$ (i) $g(x) = 2 - 3x$

## 1.2 The gradient of a straight line

The gradient (or slope) of a straight line measures its steepness. It is represented by the letter *m*.

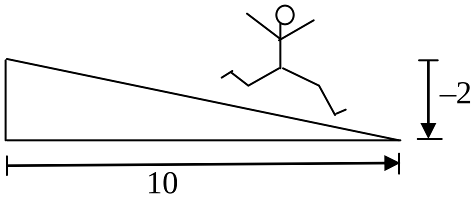
$$m = \text{gradient} = \frac{\text{rise}}{\text{run}}$$

When calculating a gradient, a "rise" may be positive, e.g.



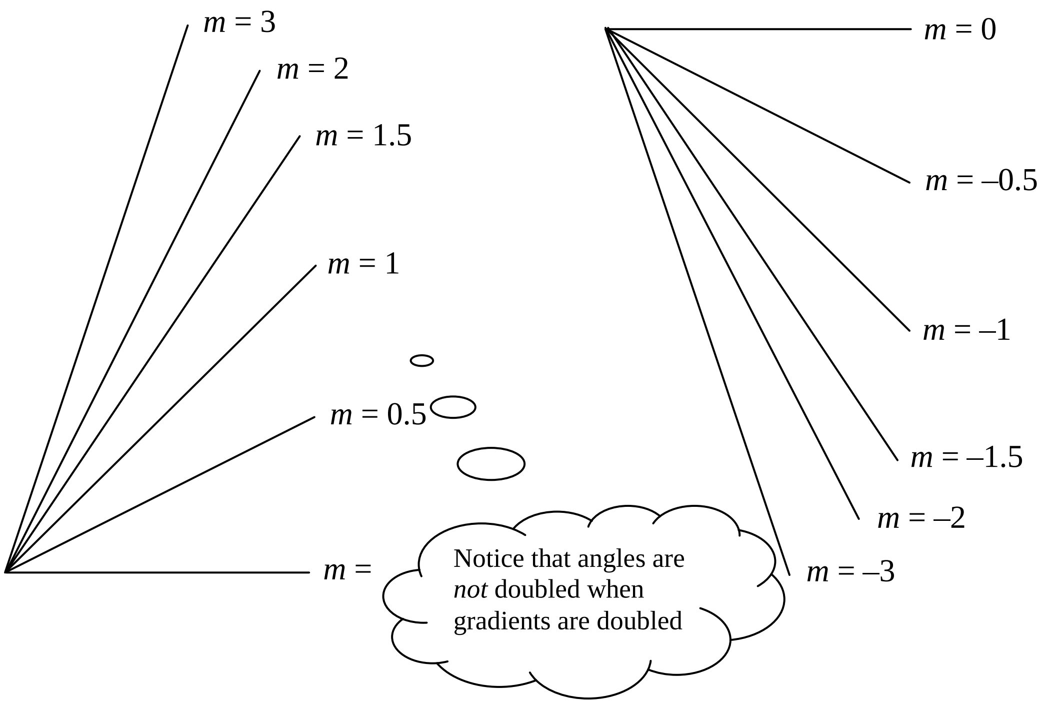

$$m = \frac{2}{10} = 0.2$$

or it may be negative, e.g.



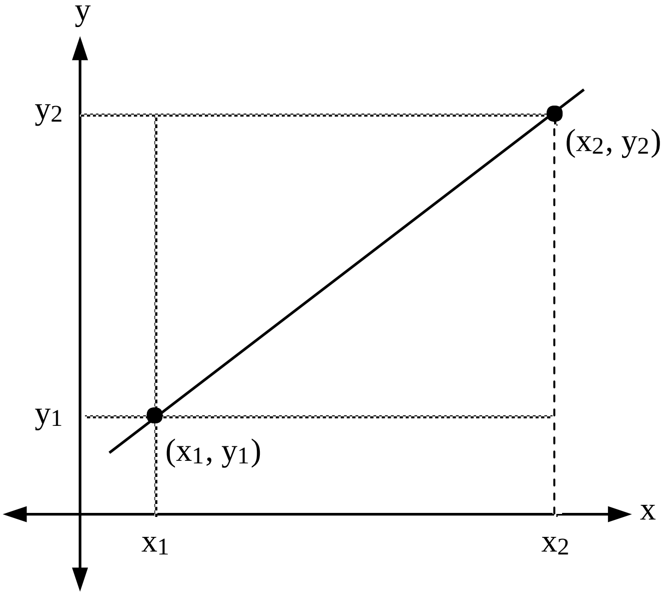

$$m = \frac{-2}{10} = -0.2$$

The steeper the line, the larger the gradient (in both the positive or negative directions):



The slope of a straight line in a graph can be calculated from *any two points* on the line.

If $(x_1, y_1)$ and $(x_2, y_2)$ are *any* two points on the line below,




then the gradient of the line is given by the formula:

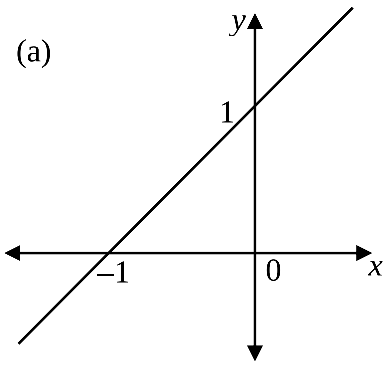
$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

*This formula is still true if the line slopes downwards, or if the two points are used in reverse order.*

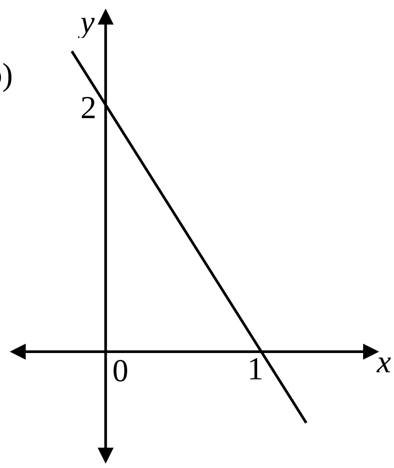
## Problems 1.2

1. Find the gradients of the lines below

(a)




(b)

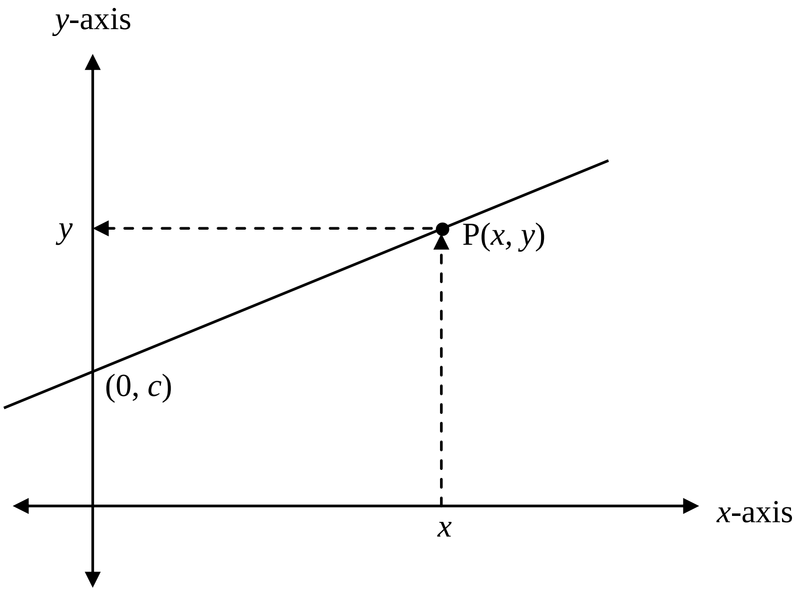



2. Draw a horizontal line and find its gradient.

## 1.3 The standard form of a straight line

The equation of a line is a formula showing the relationship between the $x$- and $y$-coordinates of any point P on the line.

Suppose a line has gradient $m$ and $y$-intercept $(0, c)$, and suppose that the point P is a general point on the line with coordinates $(x, y)$.




The diagram above shows that the gradient of the line between the $y$-intercept $(0, c)$ and the point $P(x, y)$ is

$$m = \frac{y-c}{x-0} = \frac{y-c}{x}.$$

Multiplying by $x$ gives

$$mx = y - c$$

which is the same as

$$y = mx + c$$

This formula connecting the $x$- and $y$-coordinates of the line is the equation of the line.

| |
|---|
| The equation of the straight line with gradient $m$ and $y$-intercept $(0, c)$ is $y = mx + c$. |

This equation is called the *standard form* of a straight line. Every straight line with gradient $m$ and $y$-intercept $(0, c)$ has the equation $y = mx + c$, and the graph of the equation $y = mx + c$ is a straight line with gradient $m$ and $y$-intercept $(0, c)$.

## 7 *Linear Functions*

Linear functions are functions that have straight line graphs, so:

> A linear function of $x$ has the standard form $f(x)=mx+c$.

*Example*

The graph of $y=2x-1$ is a straight line with gradient $m = 2$ and $y$-intercept (0, –1).

*Example*

The function $f(x)=-2x+3(1+x)$ is a linear function of x because it can be rewritten as $f(x)=x+3$.

Its graph has gradient $m = 1$ and $y$-intercept (0, 3).

*Example*

Find the equation of the straight line that has gradient 2 and passes through the point (3, 4).

*Answer*

As the line has gradient $m = 2$, its equation is $y=2x+c$.

To find $c$, substitute (3, 4) into the equation of the line.

$$4 = 2\times 3 + c$$

$$c = -2$$

The equation is $y=2x-2$.

*Check*

Put $x = 3$, then $y=2\times 3-2=4$.

*Example*

Find the equation of the straight line that contains the two points (1, 1) and (3, 4).

*Answer*

The gradient of the line is $m=\dfrac{y_2-y_1}{x_2-x_1}=\dfrac{4-1}{3-1}=\dfrac{3}{2}$.

> It makes no difference if (3, 4) is used instead, the calculation just looked a bit easier with (1, 1).

As the line has gradient $m=\frac{3}{2}$, its equation is $y=\frac{3}{2}x+c$.

To find $c$, substitute (1, 1) into the equation of the line.

$$1=\frac{3}{2}+c$$

$$c=1-\frac{3}{2}=-\frac{1}{2}$$

The equation is $y = \frac{3}{2}x - \frac{1}{2}$.

*Check*

Put $x = 1$, then $y = \frac{3}{2} - \frac{1}{2} = 1$.

Put $x = 3$, then $y = \frac{3}{2} \times 3 - \frac{1}{2} = 4$.

## Problems 1.3

1. What are the gradients and *y*-intercepts of the following lines?

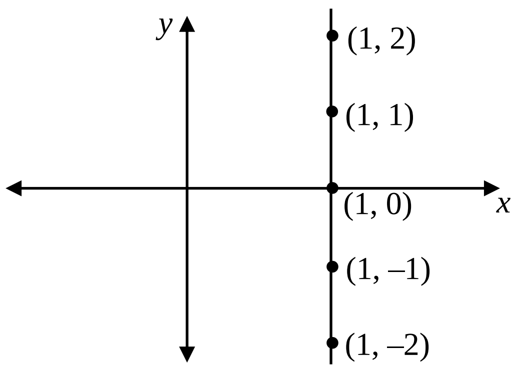
   (a) $y = 3x + 2$ (b) $y = 1 - 2x$ (c) $y = x + 6$ (d) $y = -x$.

2. Find the equation of the line that has gradient –2 and passes through (2, 3).
3. If two lines are *parallel*, then they have the same gradients or slopes.

   Find the equation of the line that is parallel to $y = 2 - x$ and passes through (–1, 2).
4. Find the equation of the line that passes through (1, –1) and (2, 3).
5. Find the *x*- and *y*-intercepts of the line that is parallel to $y = 2x + 3$ and passes through (2, 1).
6. Find the *x*- and *y*-intercepts of the line that contains the points (-2, 1) and (1, -2).
7. If two lines are *perpendicular*, then their gradients, $m_1$ and $m_2$, satisfy $m_1 \times m_2 = -1$. For example, the lines $y = 4x + 1$ and $y = -0.25x + 10$ are perpendicular. Find the equation of the line which is perpendicular to y = 2x – 4, and which has the same *y*-intercept as y = 2x – 4. (It may help to sketch the line first.)

## 1.4 The general equation of a straight line

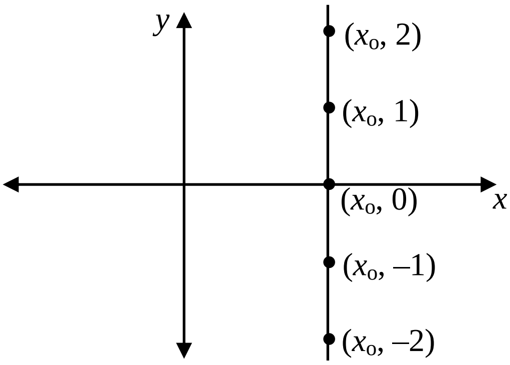
The gradient of a line, $m = \frac{y_2 - y_1}{x_2 - x_1}$, is only be defined when the denominator of the formula is not zero. This is true for all lines except vertical lines.



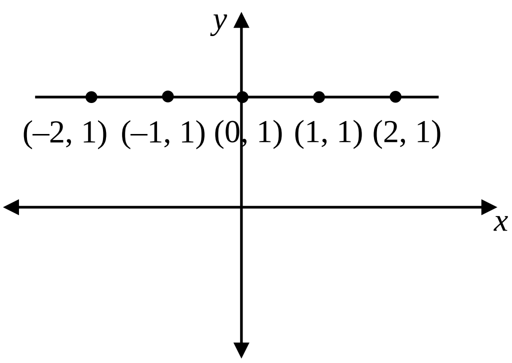

Because of this, the standard form of a straight line does not apply to vertical lines!

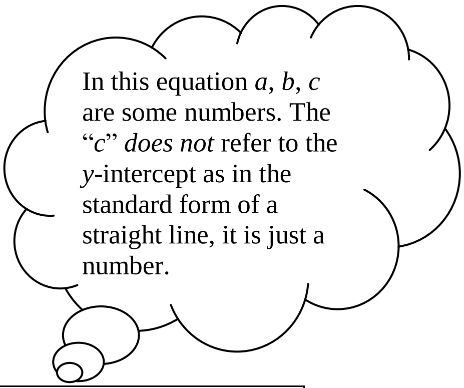
What is the equation of the vertical line above? Remember that the equation of a line is a formula that shows the relationship between the x- and y-coordinates of any point P on the line. Each point on the vertical line above has $x$-coordinate equal to 1, without any restriction on the $y$-coordinates of the points on the line. The equation describing this situation is $x = 1$ (meaning $x$-coordinate = 1).

Similarly, a vertical line with points of the form $(x_o, y)$, with no restriction on $y$-coordinates, has equation $x = x_o$.




For comparison, the graph below is the graph of the horizontal line $y = 1$, having gradient $m = 0$.




The general equation of a straight line is $ax + by = c$, where $a$, $b$ and $c$ are numbers.

This general equation includes the equations of both vertical and non-vertical lines.

*Examples*

(a) The line $y = 2x - 1$ can be written as $2x - y = 1$, with $a = 2$, $b = -1$ and $c = 1$.

(b) The vertical line $x = 2$ has $a = 1$, $b = 0$ and $c = 2$.

(c) The horizontal line $y = 5$ has $a = 0$, $b = 1$ and $c = 5$.

It is easy to calculate the intercepts of a straight line from its general equation . . .

*Example*

Find the intercepts of the line 2x + 3y = 12

*Answer*

Put y = 0, then $2x = 12 \Rightarrow x = 6$.

Put $x$ = 0, then $3y = 12 \Rightarrow y = 4$.

The intercepts are (6, 0) and (0, 4).

. . . however, to find the gradient of a line, you need to change its equation into the standard form.

*Example*

Find the gradient of 2x + 3y = 12

*Answer*

$$\begin{aligned} 2x + 3y &= 12 \\ 3y &= -2x + 12 \\ y &= -\frac{2}{3}x + 4 \end{aligned}$$

The gradient is $m = -\frac{2}{3}$.

# 2

# Simultaneous Linear Equations

It is often useful to find the *point of intersection* of two straight lines.

*Example*

A company manufactures seat covers. It leases workspace at a fixed monthly cost of \$ 2 000. Each seat cover costs \$15 to make (materials and labour). If $x$ seat covers were made each month, the total monthly cost ($C$) would be given by the cost function:

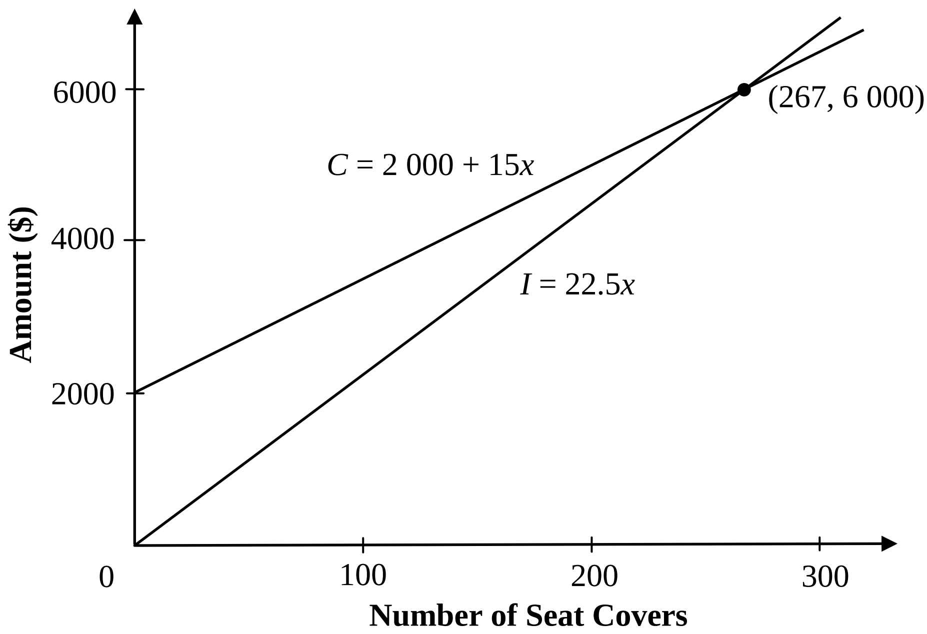
$$C = 2\,000 + 15x \qquad \text{(dollars)}$$

Each seat cover will be sold for \$22.50. The total income from sales ($I$) is given by the income function:

$$I = 22.5x \qquad \text{(dollars),}$$

where $x$ is the number of seat covers sold.

The graphs of the cost and income functions are drawn below using the same axes. The point of intersection of the two lines corresponds to the breakeven point, where income equals cost.




Two non-parallel lines have only one point of intersection, and the coordinates of this point can be estimated using a graph. Alternatively, the coordinates can be found exactly by using algebra to solve a *pair of simultaneous linear equations*.

Simultaneous equations are equations that have a shared solution.

*Example*

Solve $\begin{cases} y = 2000 + 15x \\ y = 22.5x \end{cases}$ simultaneously.

> We are solving these equations simultaneously for shared *x*- and *y*-values.

*Answer*

As the two equations have a shared solution for *x* and *y*, we can write

$$22.5x = 2000 + 15x$$
$$7.5x = 2000$$
$$x = \frac{2000}{7.5} = 266.7 \quad (4\text{ sf})$$

and $$y = 22.5x = 22.5 \times \frac{2000}{7.5} = 6000.$$

> Substituting the solution for *x* back into one of the original equations gives the solution for *y*.

There are two methods for solving a pair of simultaneous linear equations: *substitution* and *elimination*. It doesn't matter which you use.

In the following examples, the equations are written in the general form of straight lines, as this will be used in later problems.

*Example*

Solve the pair of simultaneous equations $\begin{cases} x + 2y = 4 \\ 7x - 5y = 9 \end{cases}$ by substitution.

*Answer*

$$x + 2y = 4 \quad \ldots(1)$$
$$7x - 5y = 9 \quad \ldots(2)$$

> First label the equations as (1) and (2), so you can refer to them later

Solve equation (1) for *x*

$$x = 4 - 2y$$

> We first find *x* in terms of *y* (or vice versa), then *substitute* it into the other equation. This results in an equation in one unknown, which we know how to solve.

Substitute *x* into equation (2)

$$7(4 - 2y) - 5y = 9$$
$$28 - 14y - 5y = 9$$
$$-19y = -19$$
$$y = 1$$

Substitute *y* = 1 into (1) to find *x*.

$$x + 2 \times 1 = 4$$
$$x = 2$$

The solutions are *x* = 2 and *y* = 1.

*Check*

Substitute $x = 2$ and $y = 1$ into (2).

> We found $x = 2$ by substituting $y = 1$ into equation (1), so check the answer by substituting $x = 2$ and $y = 1$ into equation (2).

LHS = $7\times 2-5\times 1=14-5=9$ RHS = 9

*Example*

Solve the pair of simultaneous equations $\begin{cases} x+2y=4 \\ 7x-5y=9 \end{cases}$ by elimination.

*Answer*

$$x+2y=4 \quad \ldots(1)$$
$$7x-5y=9 \quad \ldots(2)$$

> First label the equations as (1) and (2), so you can refer to them later

$$5x+10y=20 \quad \ldots 5\times(1)\Rightarrow(3)$$
$$14x-10y=18 \quad \ldots 2\times(2)\Rightarrow(4)$$

> Multiply equation (1) by 5 and equation (2) by 2, so that the $y$'s have coefficients of the same magnitude in (3) and (4), then *eliminate* the $y$'s

Add (3) and (4) to eliminate $y$.

$$5x+10y+(14x-10y)=20+18$$
$$19x=38$$
$$x=2$$

Substitute $x = 2$ into (1)

> Substituting into $x = 2$ into (2) would give the same final answer.

$$2+2y=4$$
$$2y=2$$
$$y=1$$

The solutions are $x = 2$ and $y = 1$.

*Check*

Substitute are $x = 2$ and $y = 1$ into (2).

> Check by substituting into the **other** equation.

LHS = $7\times 2-5\times 1=14-5=9$.

RHS = 9

*Alternative answer*

$$x+2y=4 \quad \ldots(1)$$
$$7x-5y=9 \quad \ldots(2)$$

$$7x+14y=28 \quad \ldots 7\times(1)\Rightarrow(3)$$
$$7x-5y=9 \quad \ldots 1\times(2)\Rightarrow(4)$$

> Multiply equation (1) by 7 and equation (2) by 1, so that the $x$'s have coefficients of the same magnitude in (3) and (4), then *eliminate* the $x$'s

Subtract (4) from (3) to eliminate $x$.

$$7x + 14y - (7x - 5y) = 28 - 9$$
$$19y = 19$$
$$y = 1$$

Substitute $y$ = 1 into (1)

Substituting $y$ = 1 into (2) would give the same final answer.

$$x + 2 = 4$$
$$x = 2$$

The solutions are $x$ = 2 and $y$ = 1.

Check as before.

## Problems 2

Solve the following pairs of simultaneous equations

(a) $2x + y = 12$, $2x - y = 8$

(b) $x + 2y = 6$, $x + 5y = 18$

(c) $2x - y = 4$, $4x - y = 10$

(d) $2x + y = 22$, $3x - 2y = 5$

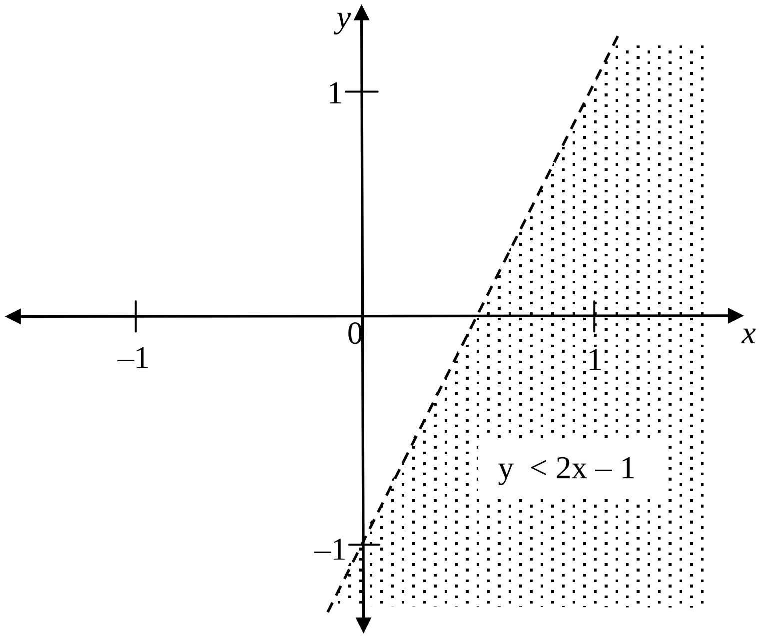
(e) $x + 3y = 8$, $3x - y = 4$

(f) $3x - 4y = 7$, $2x + 5y = -3$

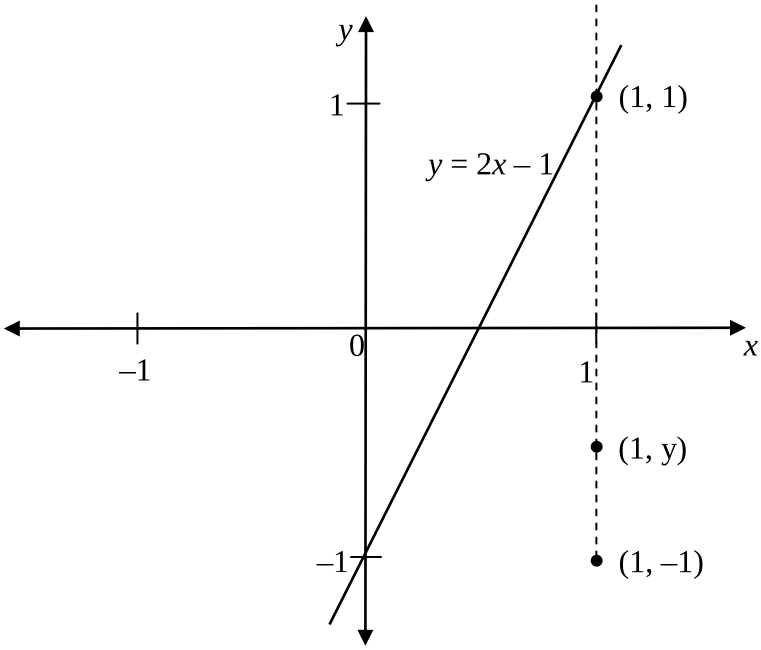
# 3

# Linear Inequalities

The inequality $y < 2x - 1$ is an example of a linear inequality. The *graph of this inequality* is the set of all points in the shaded region below. This can be written as $\{(x, y) : y < 2x - 1\}$.




To graph this linear inequality, we first draw the line $y = 2x – 1$. This line has the intercepts (0.5, 1) and (0, –1) and is shown below.

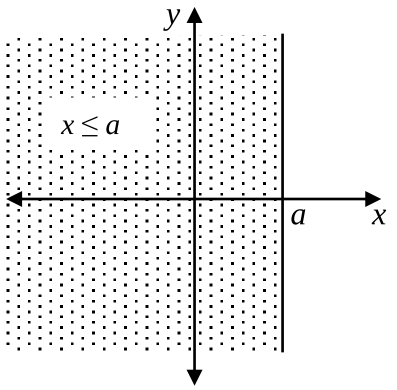


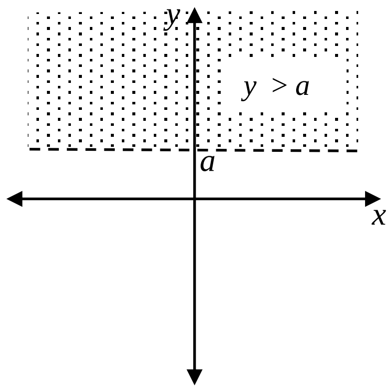
Now look at the vertical line $x$ = 1 on the graph. Begin at the point (1, –1). The $x$- and $y$-coordinates of this point satisfy the inequality $y < 2x - 1$ as $-1 < 2 \times 1 - 1$. Now move slowly upwards along the line $x$ = 1. As you move upwards, the $y$-coordinate of the point (1, $y$) will increase. Eventually you will reach the line $y = 2x - 1$ at the point (1, 1). The $x$- and $y$-coordinates of this point satisfy the equation $y = 2x - 1$. Before we reach the line $y = 2x - 1$, the $x$- and $y$-coordinates of each point on $x$ = 1 satisfied the inequality $y < 2x - 1$ (because the value of $y$ has been increasing as we moved upwards). Now continue moving upwards along line $x$ = 1. The $y$-coordinate of the point (1, $y$) will continue to increase. After you cross over the line $y = 2x - 1$, you can see that the $x$- and $y$-coordinates of each point on $x$ = 1 will satisfy the inequality $y > 2x - 1$.

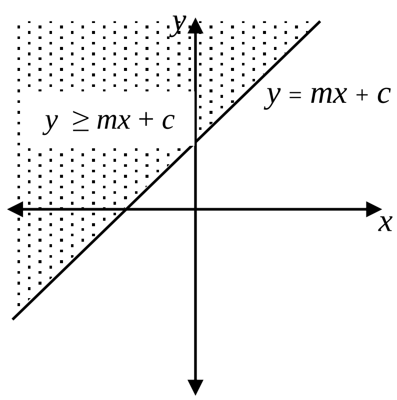
This shows that all the points on line $x$ = 1 and below $y = 2x - 1$ satisfy $y < 2x - 1$, and that all points on line $x$ = 1 and above $y = 2x - 1$, satisfy $y > 2x - 1$.

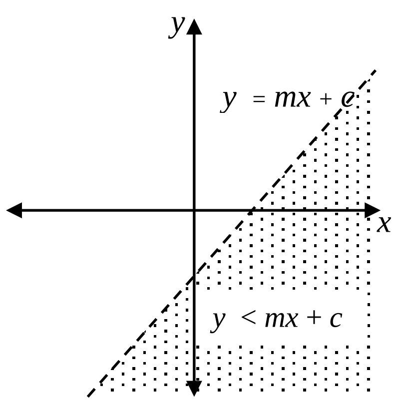
We can repeat this, beginning at any point on any vertical line. Each will show that the line $y = 2x - 1$ divides the plane into 2 regions, and that points in the region below the line satisfy $y < 2x - 1$ and all points in the region above satisfy $y > 2x - 1$. The line $y = 2x - 1$ is drawn with a dashed line in the graph of $y < 2x - 1$ to show that the points on the line do not satisfy $y < 2x - 1$.

*Examples*



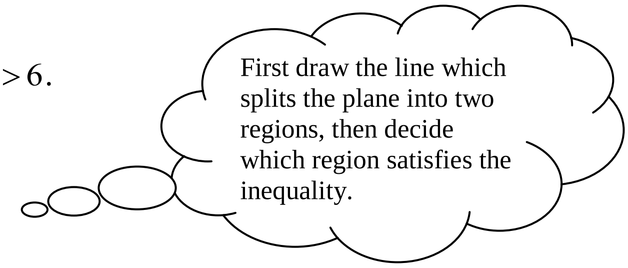



A straight line divides the coordinate plane into two regions, one below the line and one above it. If you are not sure which region satisfies a linear inequality, just select a point in one region and check if it satisfies the inequality.

*Example*

Sketch the region satisfying $2x - 3y > 6$.

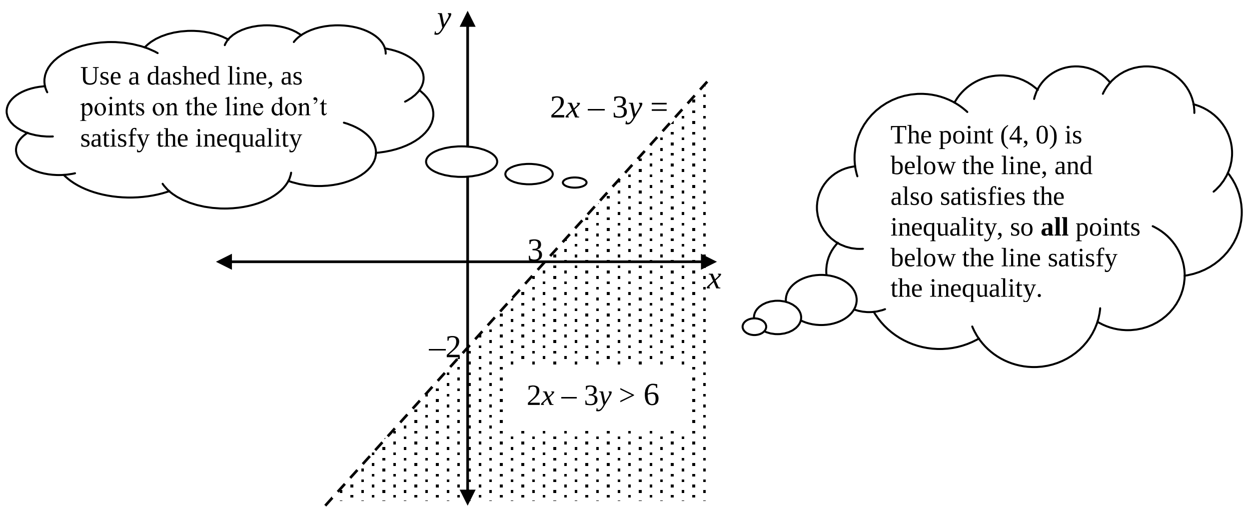



*Answer*

Put $y = 0$, then $2x = 6 \Rightarrow x = 3$.

Put $x = 0$, then $-3y = 6 \Rightarrow y = -2$.

The intercepts of $2x - 3y = 6$ are (3, 0) and (0, –2).




*Example*

A person on a certain diet should have less than 300 mg of cholesterol per day. It is known that 1 gm of whole egg contains 6.6 mg of cholesterol and 1 gm of liver contains 3.6 mg of cholesterol. Find the relationship between the quantities of egg and liver that can be allowed in the diet, assuming that these are the main sources of cholesterol. Draw a graph showing this relationship.

*Answer*

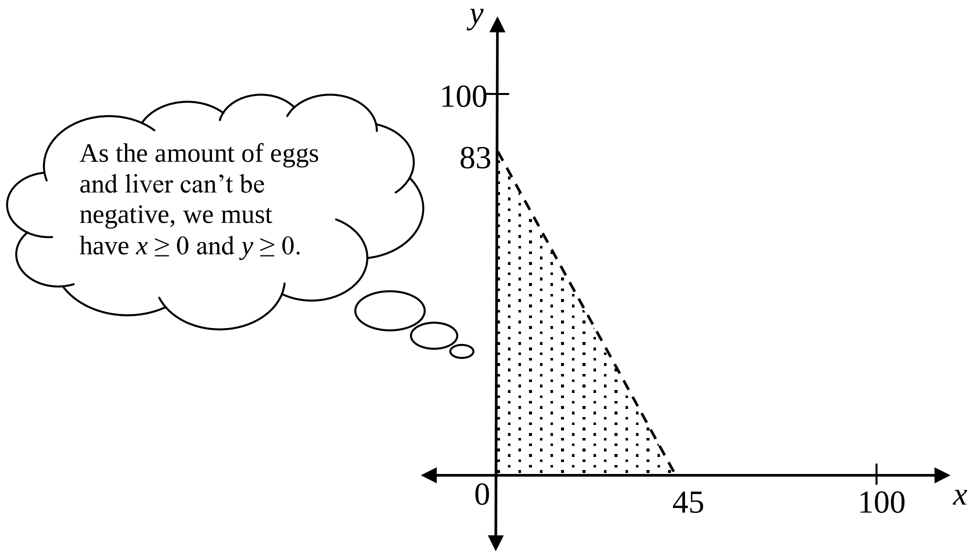
(a) The relationship.

If $x$ gm of egg and $y$ gm of liver is eaten, then the amount of cholesterol will be $6.6x + 3.6y$. This has to be less than 300, so the relationship between the quantities of egg and liver allowed in the diet is $6.6x + 3.6y < 300$.

(b) The graph of $6.6x + 3.6y < 300$.

Put x = 0, then $3.6y = 300 \Rightarrow y = \frac{300}{3.6} \approx 83$.

Put y = 0, then $6.6x = 300 \Rightarrow x = \dfrac{300}{6.6} \approx 45$.
The intercepts are (45, 0) and (0, 83).

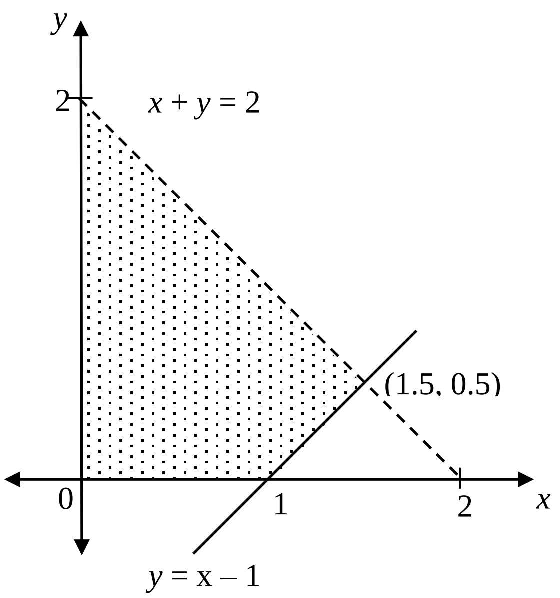



*Example*

Solve the simultaneous linear inequalities below graphically.

$$\begin{cases} x + y \le 2 \\ y > x - 1 \\ x \ge 0 \\ y \ge 0 \end{cases}$$

*Answer*



## Problems 3

1. Sketch the graphs of the following inequalities.

(a) $y > 1 - x$ (b) $2x - y \le 4$ (c) $3x \ge y - 6$

2. Solve the simultaneous linear inequalities below graphically.

$$\begin{cases} x + y > 2 \\ 3x + y < 3 \end{cases}$$

3(a) A laboratory needs at least 300 beakers of one size and at least 400 beakers of a second size. It is decided that the total number of beakers should be less than 1200. Draw a graph showing the possible numbers of each kind of beaker.

(b) Each beaker of the first size needs 45 sq cm of shelf space, and each beaker of the second size needs 30 sq cm of shelf space. However, there is only 4 sq m of shelf space. Draw a graph showing the possible numbers of each kind of beaker.

# A

# Appendix: Linear Equations

A linear equation is an equation like $2x - 3 = 12$. This refresher consists of a series of examples showing how to solve these equations.

*The general principle for solving all linear equations is to rearrange the equation so that the unknown is on the left hand side and a number is on the right hand side.*

*Example*

Solve $x - 3 = 0$

> Always carry out the same action on both sides of the equation.

*Answer*

Add 3 to both sides of the equation:

$$x - 3 = 0$$
$$x - 3 + 3 = 0 + 3$$
$$x = 3$$

> Some of these steps don't need to be written down.

*Example*

Solve $w + 10 = 0$

*Answer*

Subtract 10 from both sides of the equation:

$$w + 10 = 0$$
$$w + 10 - 10 = 0 - 10$$
$$w = -10$$

*Example*

Solve $2s + 10 = 0$

*Answer*

> Rearrange the equation with a multiple of the unknown on the LHS, and a number on the RHS.

$$2s + 10 = 0$$
$$2s = -10$$
$$s = -5$$

> Then divide both sides by 2 to get $s$ by itself.

*Example*

Solve $\frac{x}{2} - 3 = 0$

*Answer*

$$\frac{x}{2} - 3 = 0$$

$$2 \times \frac{x}{2} = 2 \times 3$$

$$x = 6$$

> Multiply both sides by 2 to get *x* by itself.

*Example*

Solve $3w + 10 = 4w + 12 + 3(1 - w)$

*Answer*

$$3w + 10 = 4w + 12 + 3(1 - w)$$

> First expand brackets and simplify each side.

$$3w + 10 = 4w + 12 + 3 - 3w$$

$$3w + 10 = w + 15$$

$$3w + 10 - w = w + 15 - w$$

> Unknowns on the LHS, numbers on the RHS.

$$2w + 10 = 15$$

$$2w = 5$$

$$w = \frac{5}{2}$$

*Example*

Solve $\frac{3}{x+1} = 2$.

*Answer*

$$\frac{3}{x+1} = 2$$

> Remove the denominator on the LHS by multiplying **both** sides by *x* + 1.

$$(x+1) \times \frac{3}{x+1} = (x+1) \times 2$$

$$3 = 2(x+1)$$

$$3 = 2x + 2$$

$$1 = 2x$$

> Interchange the LHS and RHS to get the unknown on the LHS.

$$2x = 1$$

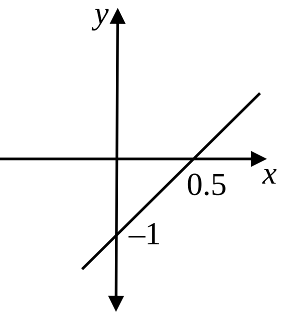
$$x = \frac{1}{2}$$

# C

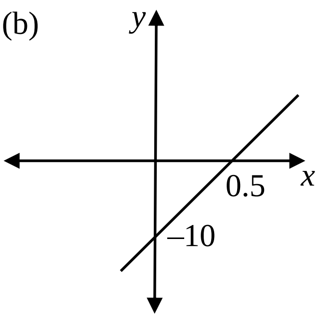
# Appendix: Answers

## Section 1.1

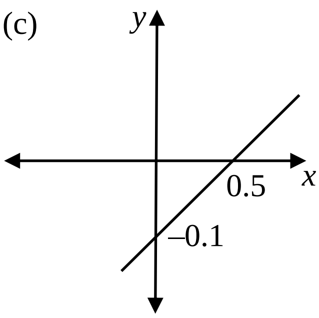
(a)



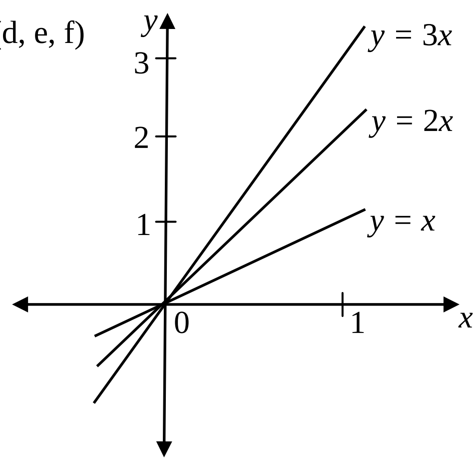

(b)



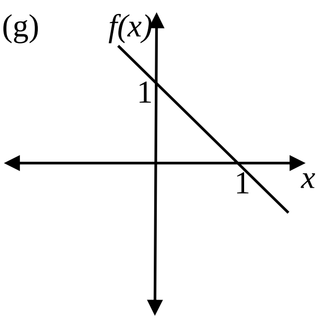

(c)



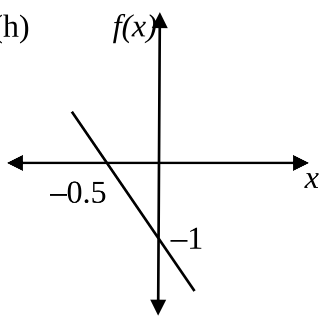

(d, e, f)



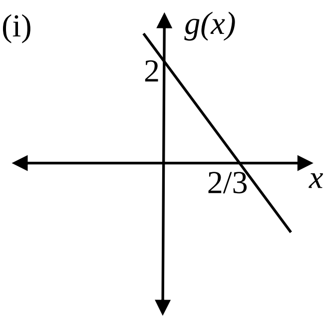

(g)




(h)




(i)

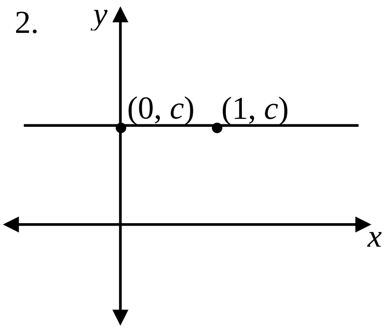


## Section 1.2

1(a) $(x_1, y_1) = (-1, 0), (x_2, y_2) = (0, 1) \Rightarrow m = \dfrac{1-0}{0-(-1)} = 1$

1(b) $(x_1, y_1) = (1, 0), (x_2, y_2) = (0, 2) \Rightarrow m = \dfrac{2-0}{0-1} = -2$

2.




$(x_1, y_1) = (0, c), (x_2, y_2) = (1, c)$

$m = \dfrac{c-c}{1-0} = 0$

## Section 1.3

1(a) 3 and (0, 2) 1(b) –2 and (0, 1) 1(c) 1 and (0, 6) 1(a) –1 and (0, 0)

2. The line has gradient $m$ = –2, so its equation is $y = -2x + c$.

   Substitute (2, 3) into the equation of the line.

   $3 = -2 \times 2 + c$

   $c = 7$

The equation is $y = -2x + 7$.

3. The line has gradient $m$ = –1, so its equation is $y = -x + c$.

   Substitute (–1, 2) into the equation of the line.

   $2 = -1 \times -1 + c$

   $c = 1$

The equation is $y = -x + 1$.

4. The gradient of the line is $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{3-(-1)}{2-1} = 4$.

As the line has gradient $m = 4$, its equation is $y = 4x + c$.

To find $c$, substitute (1, –1) into the equation of the line.

$-1 = 4 + c$

$c = -5$

The equation is $y = 4x - 5$.

5. The line has gradient $m = 2$, so its equation is $y = 2x + c$.

Substitute (2, 1) into the equation of the line.

$$1 = 2 \times 2 + c$$
$$c = -3$$

The $y$-intercept is (0, –3), and the equation is $y = 2x - 3$.

Put $y = 0$ in $y = 2x - 3 \Rightarrow x = 3/2$, so the $x$-intercept is (3/2, 0).

6. The gradient of the line is $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-2-1}{1-(-2)} = -1$.

As the line has gradient $m = -1$, its equation is $y = -x + c$.

To find $c$, substitute (-2, 1) into the equation of the line.

$$1 = -(-2) + c$$
$$c = -1$$

The $y$-intercept is (0, –1), and the equation is $y = -x - 1$.

Put $y = 0$ in $y = -x - 1 \Rightarrow x = -1$, so the $x$-intercept is (–1, 0).

7. The gradient of the line is $m = -\dfrac{1}{2}$ and the $y$-intercept is (0, –4). The equation is $y = -\dfrac{1}{2}x - 4$.

## Section 2

*These questions can be answered in more than one way.*

(a)
$$2x + y = 12 \qquad \ldots(1)$$
$$2x - y = 8 \qquad \ldots(2)$$

Solve equation (1) for $y$

$$y = 12 - 2x$$

Substitute $y$ into equation (2)

$$2x - (12 - 2x) = 8$$
$$4x - 12 = 8$$
$$x = 5$$

Substitute $x = 5$ into (1).

$$10 + y = 12$$
$$y = 2$$

The solutions are $x = 5$ and $y = 2$.

(b)
$$x + 2y = 6 \qquad \ldots(1)$$
$$x + 5y = 18 \qquad \ldots(2)$$

Solve equation (1) for $x$

$$x = 6 - 2y$$

Substitute $y$ into equation (2)

$$6 - 2y + 5y = 18$$
$$3y = 12$$
$$y = 4$$

Substitute $x = 4$ into (1).

$$x + 8 = 6$$
$$x = -2$$

The solutions are $x = –2$ and $y = 4$.

(c) $2x-y=4 \quad \ldots(1)$
$4x-y=10 \quad \ldots(2)$

Solve equation (1) for $y$

$$y=2x-4$$

Substitute $y$ into equation (2)

$$4x-(2x-4)=10$$
$$2x+4=10$$
$$x=3$$

Substitute $x$ = 3 into (1).

$$12-y=10$$
$$y=2$$

The solutions are $x$ = 3 and $y$ = 2.

(d) $2x+y=22 \quad \ldots(1)$
$3x-2y=5 \quad \ldots(2)$

$4x+2y=44 \quad \ldots 2\times(1)\Rightarrow(3)$
$3x-2y=5 \quad \ldots \quad (2)\Rightarrow(4)$

Add (3) and (4) to eliminate $y$.

$$4x+2y+(3x-2y)=44+5$$
$$7x=49$$
$$x=7$$

Substitute $x$ = 7 into (1)

$$14+y=22$$
$$y=8$$

The solutions are $x$ = 7 and $y$ = 8.

(e) $x+3y=8 \quad \ldots(1)$
$3x-y=4 \quad \ldots(2)$

$x+3y=8 \quad \ldots \quad (1)\Rightarrow(3)$
$9x-3y=12 \quad \ldots 3\times(2)\Rightarrow(4)$

Add (3) and (4) to eliminate $y$.

$$x+3y+(9x-3y)=8+12$$
$$10x=20$$
$$x=2$$

Substitute $x$ = 2 into (1)

$$2+3y=8$$
$$y=2$$

The solutions are $x$ = 2 and $y$ = 2.

(f) $3x-4y=7 \quad \ldots(1)$
$2x+5y=-3 \quad \ldots(2)$

$6x-8y=14 \quad \ldots 2\times(1)\Rightarrow(3)$
$6x+15y=-9 \quad \ldots 3\times(2)\Rightarrow(4)$

Subtract (3) from (4) to eliminate $x$.

$$6x+15y-(6x-8y)=-9-14$$
$$23y=-23$$
$$y=-1$$

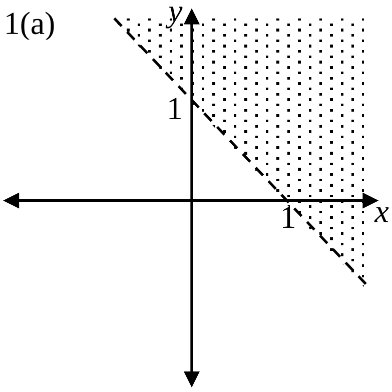
Substitute $y$ = -1 into (1)

$$3x-4\times(-1)=7$$
$$3x+4=7$$
$$x=1$$

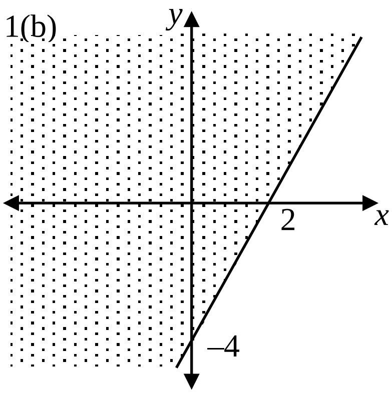
The solutions are $x$ = 1 and $y$ = -1.

**Section 3**

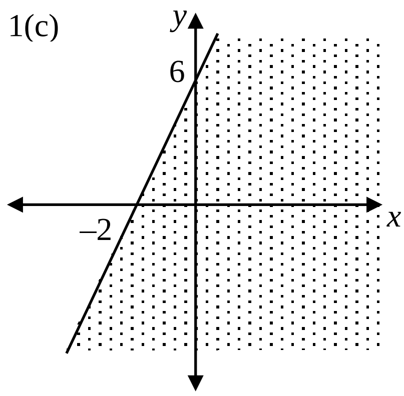
1(a)



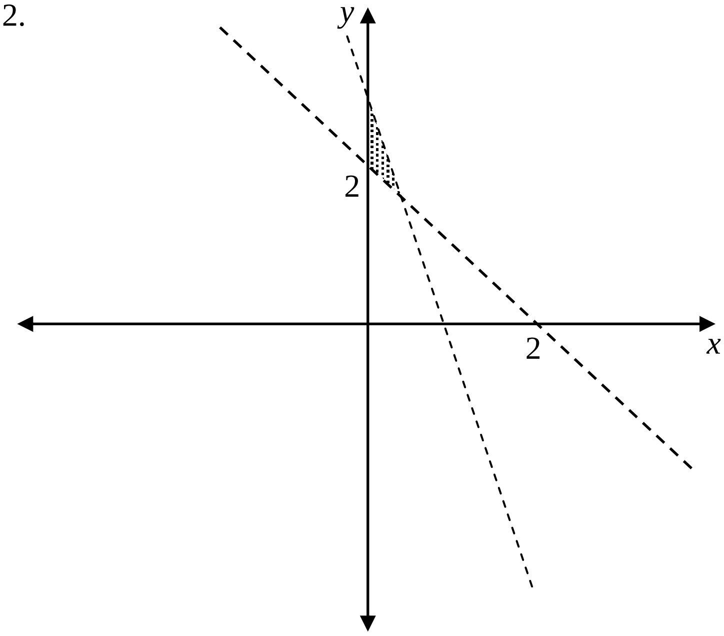

1(b)



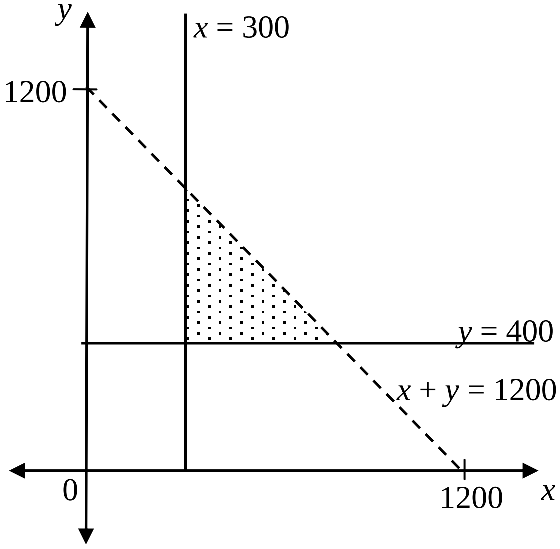

1(c)




2.




3(a) If the laboratory obtained $x$ beakers of the first size and $y$ beakers of the second size, then we should have $x \geq 300$, $y \geq 400$ and $x + y \leq 1200$. The shaded area below shows the graph of these inequalities.



3(b) If shelf space is taken into account, then we need to have $45x + 30y \leq 40\ 000$. This additional inequality is included in the graph below.